Acoustic modeling using a three-dimensional coupled-mode model

Megan S. Ballard
Applied Research Laboratories at the University of Texas at Austin
Environmental Science Laboratory
Austin, Texas 78758
phone: (512) 835-3953 fax: (512) 490-4225 email: meganb@arlut.utexas.edu

LONG-TERM GOALS

The over-all goal of this research is the development of an accurate and reliable propagation model applicable to environments which exhibit strong range dependence in all three spatial dimensions.

OBJECTIVES

The objective of this work is to gain an understanding the physics of propagation in continental shelf areas, specifically horizontal refraction and mode coupling induced by three-dimensional (3D) inhomogeneities in the waveguide. A coupled-mode approach has been applied for this purpose. The coupled-mode approach is attractive for solving problems involving 3D propagation for several reasons. First, this technique provides intuitive results for understanding the features responsible for observed propagation effects in range-dependent environments. For example, upslope propagation is characterized by acoustic energy radiated into the bottom at discrete depths associated with mode cut-off. The modal decomposition of the acoustic field has also been used to describe horizontal refraction in a wedge-shaped ocean, for which the single-mode interference pattern associated with rays launched up and across the shelf has been well documented [Weinberg and Burridge (1974)]. Furthermore, coupled-mode solutions are highly accurate and have been used for benchmarking solutions to range-dependent problems [Jensen and Ferla (1990)]. In order to appreciate the limitations of existing 3D models, it is necessary to have methods which can provide reference solutions for comparison.

APPROACH

A 3D acoustic propagation model based on the stepwise coupled-mode approach [Evans (1983)] implemented as a single-scatter solution has been developed. This technique is based on a hybrid modeling approach for which normal modes are applied in the vertical dimension and a PE solution is applied in the horizontal dimension.

The inhomogeneous Helmholtz equation for pressure $P(r, \theta, z)$ at range r, azimuth θ, and depth z from a

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point continuous wave source of amplitude $S(\omega)$ located at range $r = 0$ and depth $z = z_0$, is given by

$$\rho(r,\theta,z)\nabla \cdot \left[\frac{1}{\rho(r,\theta,z)}\nabla P(r,\theta,z)\right] + k^2(r,\theta,z)P(r,\theta,z) = -4\pi S(\omega)\frac{\delta(r)}{r}\delta(z-z_0), \qquad (1)$$

where $k(r,\theta,z) = \omega/c(r,\theta,z)$ is the acoustic wavenumber, $\omega = 2\pi f$, f is the acoustic frequency, $c(r,\theta,z)$ is sound speed, and $\rho(r,\theta,z)$ is density.

The solution for pressure is found by a separation of variables

$$P(r,\theta,z) = \sum_{m=1}^{M} A_m(r,\theta)\phi_m(z;r,\theta), \qquad (2)$$

where $A_m(r,\theta)$ are the modal amplitudes, and $\phi_m(z;r,\theta)$ are the modal eigenfunctions.

The modal eigenfunctions $\phi_m(z;r,\theta)$ satisfy

$$\rho(r,\theta,z)\frac{\partial}{\partial z}\left[\frac{1}{\rho(r,\theta,z)}\frac{\partial\phi_m(z;r,\theta)}{\partial z}\right] + \left[k^2(r,\theta,z) - k_m^2(r,\theta)\right]\phi_m(z;r,\theta) = 0, \qquad (3)$$

where $k_m(r,\theta)$ is the horizontal wavenumber of the mth mode. Boundary conditions are defined by the plane wave reflection coefficient below the upper halfspace and above the lower halfspace. In this work, the Pekeris branch cut is chosen such that the total pressure is calculated from a Pekeris branch line integral, plus a finite sum of trapped modes, plus an infinite sum of leaky modes. A small gradient is introduced in the lower halfspace which effectively removes the branch point and associated branch cut from the problem [Westwood and Koch (1999)]. As a result, the leaky modes eventually decay as a function of depth in the lower half space. The eigenfunctions are normalized so that

$$\int_0^\infty \frac{1}{\rho(r,\theta,z)}\phi_m(z;r,\theta)\phi_n(z;r,\theta)dz = \delta_{mn}. \qquad (4)$$

In the absence of mode coupling, the adiabatic solution for the mode amplitudes $\bar{A}_m(x,y)$ satisfies

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\bar{A}_m}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\bar{A}_m}{\partial\theta^2} + k_m^2(r,\theta)\bar{A}_m = -4\pi S(\omega)\frac{\delta(r)}{r}\frac{\phi_m(r_0,z_0)}{\rho(r,\theta,z_0)}. \qquad (5)$$

This equation must be solved for each mode with the horizontal refraction determined by the modal phase speed $c_{ph_m}(x,y) = \omega/\text{Re}\{k_m(x,y)\}$ and modal attenuation $\alpha_m(x,y) = \text{Im}\{k_m(x,y)\}$. Such a two dimensional (2D) Helmholtz equation can be solved by standard techniques. In this work, the solution to the horizontal refraction equation (Eq. (5)) is obtained from a PE model [Collins (1994)] which has been modified for cylindrical coordinates [Napolitano (1985)],

$$\frac{\partial\bar{A}_m}{\partial r} = ik_0\sqrt{1 + k_0^{-2}\left(\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + k^2 - k_0^2\right)}\bar{A}_m, \qquad (6)$$

where $k_0 = \frac{\omega}{c_0}$ is the reference wavenumber, and c_0 is the reference sound speed.

Mode-coupling is incorporated into the PE solution by a stepwise coupled-mode technique [Evans (1983)]. This approach, originally derived for the 2D Helmholtz equation, is applied by discretizing a range-dependent environment into a series of range-independent segments for which continuity of

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pressure and particle velocity must be satisfied at the vertical boundaries between segments. Application of the boundary conditions results in a set of linear equations from which the coupling matrix $R_{m,n}(r,\theta)$ is obtained,

$$R_{m,n}(r,\theta) = C_{m,n}^{RL}(r,\theta) + C_{m,n}^{LR}(r,\theta)\frac{k_n(r_L,\theta)}{k_n(r_R,\theta)}, \tag{7}$$

where

$$C_{m,n}^{LR}(r,\theta) = \int_0^\infty \frac{1}{\rho(r_L,\theta,z)}\phi_m(z;r_L,\theta)\phi_n(z;r_R,\theta)dz \tag{8a}$$

$$C_{m,n}^{RL}(r,\theta) = \int_0^\infty \frac{1}{\rho(r_R,\theta,z)}\phi_m(z;r_L,\theta)\phi_n(z;r_R,\theta)dz, \tag{8b}$$

where the suffixes L and R denote the properties to the left and right of a vertical interface. This form of the mode-coupling matrix describes the single-scatter approximation, such that each pair of segments is treated as an individual problem, thus neglecting higher-order terms resulting from multiple scattering at other interfaces. It can be shown that in the absence of mode coupling, the mode-coupling matrix reduces to the identity matrix, and the adiabatic solution is obtained.

In the 3D coupled-mode model, the range-independent segments are defined in the radial direction, forming angular sectors in the range-bearing plane. By incorporating the mode-coupling matrix into Eq. (6), the solution for the coupled-modal amplitudes is obtained,

$$\frac{\partial A_m}{\partial r} = \sum_{n=1}^N R_{m,n}(r,\theta)A_n + ik_0\sqrt{1 + k_0^{-2}\left(\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + k^2 - k_0^2\right)}A_m. \tag{9}$$

According to this solution technique, at each range step, mode-coupling is calculated by the first term in Eq. (9) and horizontal refraction is calculated by the second term. Although mode-coupling occurs only in the radial direction, the coupled energy is refracted out of the range-depth plane by the second term.

WORK COMPLETED

The main accomplishments of 2012 include: (1) a comparison of the 3D coupled-mode solution to a reference solution obtained with semi-analytic technique for propagation around a shallow-water seamount, (2) the addition of a 3D rough sea surface in the model with comparison to solution calculated with a 3D PE model, and (3) an implementation of parallel processing to allow for efficient calculations at higher frequencies and longer ranges.

RESULTS

Propagation around a conical seamount

The 3D coupled-mode model described above is applied to calculate propagation around a conical seamount in shallow water. This environment was chosen because the shallow-water seamount induces both strong horizontal refraction and mode-coupling effects. Additionally, a semi-analytic solution is available for comparison [Luo and Schmidt (2009)]. The waveguide is 250 m deep, bounded above by a flat sea surface and below by a penetrable bottom. The water column sound speed, density, and attenuation are 1500 m/s, 1.0 g/cm^3, and 0.0 dB/λ. The seamount rises to a height of 100 m above the
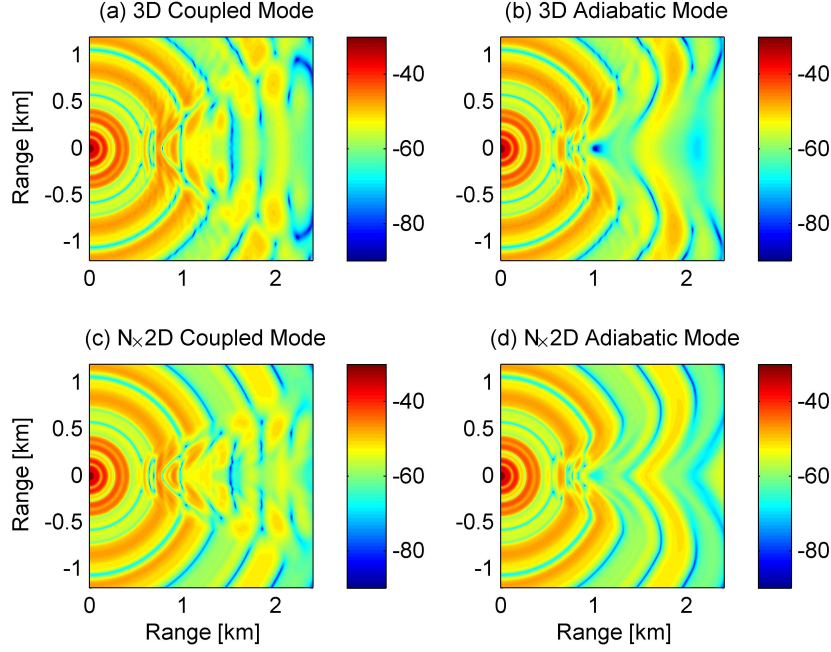
Figure 1: (Color online) Transmission loss at a depth of 100 m calculated by 3D (a) coupled- and (b) adiabatic-mode techniques and by N×2D (c) coupled- and (d) adiabatic-mode techniques.

seafloor and has a radius of 350 m at its base. The seabed sound speed, density, and attenuation are 1800 m/s, 2.0 g/cm^3, and 0.1 dB/λ. The halfspace includes an attenuation gradient such that $\Delta\alpha = \alpha_{3\lambda} - \alpha_{HS} = 0.01$ dB/λ, where α_{HS} is the attenuation of the halfspace, and $\alpha_{3\lambda}$ is the attenuation three wavelengths into the halfspace. The acoustic source is located 800 m from the axis of the seamount at a depth 100 m. The solution was calculated for an excitation frequency of 40 Hz. At this frequency, there are seven trapped modes at the location of the source. Adding five leaky modes was sufficient to obtain a convergent solution.

Transmission loss at a depth of 100 m calculated by four different approximations is shown in Figure 1. The effects of horizontal refraction are assessed by comparing results from 3D and N×2D models, which neglect out-of-plane effects by applying a 2D model to predict TL for vertical slices through the environment [Perkins and Baer (1982)]. The effects of mode-coupling are assessed by comparing adiabatic- and coupled-mode solutions.

The importance of mode-coupling is observed from the differences in the coupled- and adiabatic-mode solutions shown in Figure 1(a) and (b). In these figures, there are significant differences in the acoustic field in the region on the far side of the seamount. The effects of horizontal refraction can be understood by comparing the 3D to the N×2D solutions. For the coupled-mode solutions (Figure 1(a) and (c)), differences in the modal interference patterns are observed behind the seamount. In addition, the 3D solution is characterized by a wider diffractive pattern behind the seamount, as sound is refracted away from the seamount and into deeper water. The effects of horizontal refraction can be viewed even more clearly in the adiabatic-mode solutions (Figure 1(b) and (d)). For this case, the 3D solution shows higher loss directly behind the seamount, evidence of sound being refracted away from the seamount. Finally, both 3D solutions show fringes on the outer edges of the affected area. These result from single-mode interference patterns between refracted and non-refracted acoustic paths.

To evaluate the accuracy of the 3D coupled-mode solution, it is compared with a published result calculated using a semi-analytic technique [Luo and Schmidt (2009)]. According to this method, the conical seamount is divided into a number of rings, in each of which a series expansion of the acoustic pressure in terms of normal modes and an azimuthal Fourier series is applied. The 3D coupled-mode solution (Figure 2(a)) shows excellent quantitative agreement with the published result (see Figure 6(c) of [Luo and Schmidt (2009)]). The features described for the 3D coupled-mode solution can also be observed in the semi-analytic solution by the wider shadow behind the seamount and by the interference fringes flanking the affected region.

Propagation under a rough sea surface

To demonstrate the capability of the 3D coupled-mode model to calculate TL under a rough sea surface, an environment with a 2D sinusoidal sea surface is considered. Scattering from 2D sinusoidal surfaces has been studied extensively for the 2D propagation problem, for which the surface is composed of curved, axisymmetric wavefronts, and more recently for the 3D propagation problem, for which the surface is composed of planar wavefronts. The solution from the 3D coupled-mode model is compared to results calculated by a 3D wide angle PE (WAPE) model [Smith (2012)].

For the problem considered in this work, the height of the sea surface is described by

$$s(x) = A \sin(\frac{2\pi x}{\lambda_{surf}} + \frac{\pi}{2}) \tag{10}$$

where $A = 10$ m is in the amplitude of the surface fluctuations, and $\lambda_{surf} = 200$ m is the wavelength of the fluctuations. The surface fluctuations form corrugations in the horizontal plane which are independent of the y-direction. The mean water depth is 100 m. The water column sound speed, density, and attenuation are 1500 m/s, 1.0 g/cm^3, and 0.0 dB/λ. The seabed sound speed, density, and attenuation are 1700 m/s, 1.5 g/cm^3, and 0.15 dB/λ. The halfspace includes an attenuation gradient such that $\Delta\alpha = \alpha_{3\lambda} - \alpha_{HS} = 0.01$. Transmission loss was calculated for a 1 kHz source located at 30 m water depth. The solution was calculated from a sum of 99 modes. Including additional higher-order modes in the solution (up to 199 modes) was found to have no significant effect on the solution beyond a range of 0.1 km.

As shown in Figure 2, there are significant differences in the 3D coupled- and adiabatic-mode solutions, indicating the importance of mode-coupling effects in the solution. As observed from Figure 2(a) the coupled-mode solution shows vertical refraction of sound under the first wave crest, whereas the adiabatic-mode solution (Figure 2(c)) fails to capture scattering in the vertical plane. Further differences are apparent in the range-bearing plots (Figure 2(b,d)) where the interference patterns differ, especially around $x = \pm 0.2$ km where the vertical scattering of sound under the first wave trough occurs in the coupled-mode solution.

The 3D coupled-mode solution (Figure 2(a,b)) shows many of the same features as the solution from the 3D WAPE approximation (see Figure 2 of [Smith (2012)]). For example, both the coupled-mode and PE solutions are able to capture the vertical scattering caused by interaction with the rough sea surface. Additionally, in the range-bearing slice at 30 m depth, both solutions show interference patterns which form oblong rings near the source and transition to parabolic-shaped features for distances greater than $y = 0.1$ km. The aforementioned line of interference patterns centered around around $x = \pm 0.2$ km is also visible in both solutions. However, there are some significant differences in the solutions obtained by the two techniques. The result calculated by the 3D WAPE model predicts significantly lower TL
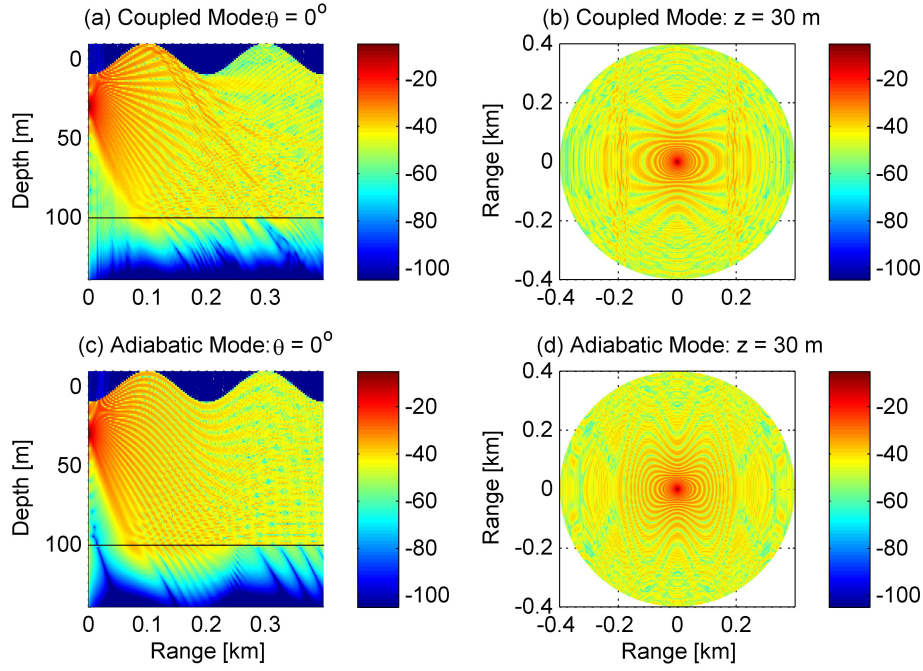
Figure 2: (Color online) Transmission loss under the sinusoidal sea surface from (a,b) coupled- and (c,d) adiabatic-mode solutions: (a,c) range-depth slice of TL along single radial (perpendicular to surface wave displacement); (b,d) range-bearing slice at 30 m depth.

than the N×2D solution (see Figure 1 of [Smith (2012)]) in the direction perpendicular to the wave crests. On the other hand, the 3D coupled-mode solution contains small differences (less than 1 dB at most depths and ranges) from the N×2D solution along the *x*-axis. These differences are evident in the range-bearing dependence as well, especially for ranges greater than 0.2 km.

High Performance Computing Capabilities

Coupled-mode codes are intrinsically computationally intensive. Efficient computation of the solution was achieved by parallel modal computations, which are performed simultaneously over azimuth at each range step. Message Passing Interface (MPI) was implemented in the code for this purpose. Computation time is proportional to the number of processors utilized by the computation. The result is a significant reduction in computation time, such that parallelized computations can be completed hundreds of times faster than serial computations. This has made it possible to compute solutions problems involving higher frequencies and longer ranges. The parallelized code is highly portable and has been run on multiple platforms including the Sun Constellation Linux Cluster Ranger [The University of Texas at Austin (2012)] and the U.S. Air Force Research Laboratory (AFRL) DoD Supercomputing Resource Center (DSRC) [U.S. Air Force Research Laboratory (2012)].

IMPACT/APPLICATIONS

The impact of this work will be an increased understanding of acoustic propagation through complicated coastal environments for which the bathymetry, seabed properties, and oceanography can vary in three dimensions.

TRANSITIONS

The primary transition for this project is an accurate and reliable model for acoustic propagation in environments with strong three-dimensional range dependence. Because coupled-mode approaches are computationally intensive, they have historically been used to benchmark faster techniques which approximate the solution to the wave equation.

RELATED PROJECTS

Geoacoutic inversion in three-dimensional environments

The goal of this project is to estimate water column sound speed in a 3D volume using modal travel time measurements from multiple source-receiver pairs. A thorough understanding of the forward problem, including the effects of horizontal refraction and mode coupling, is necessary to successfully estimate environmental parameters in regions with 3D inhomogeneities. This project is funded by ARL:UT's IR&D program.

Acoustic propagation modeling for diver detection sonar systems

The purpose of this work is to characterize waveforms at virtual receiver distances on the order of a 1000 meters away from active diver detection sonar systems installed at fixed locations within operational sites of interest. The 3D coupled-mode model is applied for this purpose. This project is funded by Space and Naval Warfare Systems Center Pacific.

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PUBLICATIONS

Refereed Journal Articles

Megan S. Ballard,"A three-dimensional propagation model with stepwise coupled modes," *Journal of the Acoustical Society of America*. [submitted]

Megan S. Ballard, Ying-Tsong Lin, and James F. Lynch,"Horizontal refraction of propagating sound due to seafloor scours over a range-dependent layered bottom on the New Jersey shelf," *Journal of the Acoustical Society of America*, **131**(4), pp. 2587-2598, 2012.

Megan S. Ballard,"Modeling three-dimensional propagation in a continental shelf environment," *Journal of the Acoustical Society of America*, **131**(3), pp. 1969-1977, 2012.

Conference Proceedings

Megan S. Ballard,"Acoustic propagation modeling in environments which induce horizontal refraction and mode coupling," *Acoustics 2012 Hong Kong*, 2012.

Chris M. Bender, Megan S. Ballard, and Preston S. Wilson, "Estimates of the water column sound-speed field in three dimensions using a linearized perturbative technique," *Proceedings of Meetings on Acoustics*, Vol. 14, pp. 070006, 2012.

Presentations

Megan S. Ballard, "Acoustic propagation modeling in environments which induce horizontal refraction and mode coupling," 163rd Meeting of the Acoustical Society of America, Hong Kong, China, May, 2012.

Megan S. Ballard, "A three-dimension propagation model using stepwise coupled modes," 162nd Meeting of the Acoustical Society of America, San Diego, California, November, 2011.

Chris M. Bender, Megan S. Ballard, and Preston S. Wilson, "Estimates of the water column sound-speed field in three dimensions using a linearized perturbative technique," 162nd Meeting of the Acoustical Society of America, San Diego, California, November, 2011.

HONORS/AWARDS/PRIZES

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